In the Redding et al. papers (although Redding is not the first author) the utility of a worker choosing as residential location and as work location can be summarized as:

In this equation denotes utility, the attractiveness of the residential location (determined by amenities, the price of housing), the attractiveness of the work location (determined by amenities, the wage), the travel cost, the distance (or travel time), while is an idiosyncratic term (a random draw from a given distribution, for each combination of and and each household an independent draw is taken).

It is assumed that the ’s are drawn from the Extreme Value type I distribution, implying that the choice probabilities are goven by the multinomial logit model. (The authors avoid these terms and talk instead about the Frechet distribution, but this is equivalent, although less familiar to transportation economists.)

We assume a population of given size, say 1. All workers have to live somewhere and work somewhere. Let be the probability that the combination is chosen. The sum of these probabilities naturally adds up to 1. In the short run, the shares of employment (let’s call them ) and the housing stock (let’s call them ) are given. Hence we have the constraints:

They can be rewritten as:

The sharea of employment and housing are given, as are the distances/travel times and (I assume) the travel cost . We can solve the model iteratively by choosing starting values for the s, plugging them into the first of the last pai of equations to get new s, which can be plugged into the second equation to get a new set of s. We can arbitrarily choose one and one .

In the spreadsheet, I use a simple example with 10 locations. The spreadsheet exercise suggest that convergence is reached soon, but we can of course use GAUSS or Python or other code to formalize it.

The important point is that we can compute equilibria under different values of the travel cost parameter . So we can see how the allocation of workers over jobs and houses changes if we increase variable car costs. This change is realized through moving houses and jobs, and therefore embodies the relocation effect of changes in car costs.

This gravity model of commuting is used in the two Redding et al. papers and, I expect, in many more quantitative economic geography papers. Since it is multinomial logit, it has the IIA property, which is always a reason for concern in transportation models, but completely ignored in this literature. If you want, you can extend the model to less restrictive discrete choice. You can also introduce mode choice. The model presented here is than the upper part of a nested logit model in which the nests refer to the various modes that can be used in travelling from to . See the paper by Ismir Mulalic and me.

Calculations one by one:

First and are given. is given in a matrix and is a constant

Iterate like the following until they don’t change much anymore. We do this to get the attractiveness of work location and residence location.

Verify, that and are as given by seeing if the probabilities add up:

Calculate the commutes (or the distance multiplied by the probability that someone has a specific pair of employment and residence)

This gives us the average commuting distance. Why? Because each pair represents a share of the whole population. If the “shares” instead were absolute numbers in terms of population, then we would have had the total commute distance. The would then represent the total commute distance from that pair, but in this example, it represents the share of the average commute. If you then multiply this with the population, you would have the total commute.